Università degli Studi di Roma Tre - PhD Course

Title: *Mathematical Quantum Mechanics*

Lecturer: Davide Fermi

Academic Year: 2022/2023

Period: November-December 2022

Duration: 20 hours (approximately)

Schedule: Tuesday 29/11, 8.00-10.00; Wednesday 30/11, 16.00-18.00; Thursday 01/12, 10.00-12.00;
Tuesday 06/12, 8.00-10.00; Wednesday 07/12, 16.00-18.00;
Tuesday 13/12, 8.00-10.00; Wednesday 14/12, 16.00-18.00; Thursday 15/12, 10.00-12.00;
Tuesday 20/12, 8.00-10.00; Wednesday 21/12, 16.00-18.00.

Place: Università Roma Tre, Dipartimento di Matematica e Fisica, L.go S.L. Murialdo 1, Aula M4

Abstract. The first part of the course will present a rigorous analysis of solvable one-body problems in non-relativistic quantum mechanics. More precisely, the spectral and dynamical features of the following models will be examined: free motion in $\mathbb{R}^d$, harmonic trap, hydrogen atom, zero-range potentials in dimension $d \leqslant 3$. The second part of the course will provide an introduction to mathematical scattering theory. The time-dependent and time-independent approaches will be outlined, addressing the existence and (asymptotic) completeness of wave operators. The definition of the scattering operator and its connection to physical cross sections will be also discussed.

Prerequisites: basic knowledge of Lebesgue theory and L^p spaces, Schwartz distributions and Sobolev spaces, functional analysis and operators in Hilbert spaces.

Exam: exposition and discussion of a research paper or textbook chapter.

Additional information: borrowed from the course “FM450 - Aspetti Matematici della Meccanica Quantistica” (Mathematical aspects of Quantum Mechanics) for the master degree in Mathematics. Live streaming available (via Microsoft Teams). Whoever is interested in attending the course should send an email to `davide.fermi@polimi.it`

Program

- **Part I - Solvable models (12 hours)**

  1. *Preliminaries* (2 hours). Compact operators and Weyl's criterion for the essential spectrum. Strongly continuous one parameter unitary groups and their generators. Stone theorem. Quantum dynamics, Schrödinger and Heisenberg representations. Weyl's representation of the canonical commutation relations.
  2. *Free particle* (2 hours). Self-adjointness domain, absolutely continuous spectrum, spectral projectors, integral kernels of resolvent operator and unitary evolution. The basic dispersive estimate. Wave packets dynamics, time evolution of average position and momentum, interference phenomena for coherent superpositions.
  3. *Harmonic oscillator* (2 hours). Creation/annihilation operators, pure point spectrum, hints at Rellich criterion for Hamiltonians with pure point spectrum, self-ajointness domain, time evolution of average position and momentum. Generalization to d-dimensional oscillator.
  4. *Hydrogen atom* (6 hours). Center of mass decomposition. Self-adjointness domain and lower boundedness (Sobolev and Kato lemmas). Ground state estimate (Hardy inequality), essential spectrum and point spectrum characterization. Angular momentum and radial problem. Bound states and eigenvalues degeneracy.

- **Part II - Elements of scattering theory (8 hours)**

  1. *Basic concepts.* Classical scattering for the Coulomb potential. General definitions of bound states and scattering states in quantum mechanics. Wave and scattering operators, transition operator and differential cross section. Completeness and asymptotic completeness of wave operators. Invariance principle.
  2. *Results on wave operators.* Cook's and Pearson's time dependent criteria. Kato-Rosenblum and Birman-Kuroda trace class methods. Applications to potential scattering.
  3. *Perturbation expansions.* Dyson series, eigenfunctions expansion and Lippmann-Schwinger equation.