MATHEMATICS OF QUANTUM MECHANICS

Subjects for the oral exams

1. Kato-Rellich theorem [Te18, §4.5], [RS81, Vol. II, §X.2, Thm. X.12 and X.15].
2. Continuous spectrum and RAGE theorem [Am09, Prop. 5.7 and Prop. 5.9].
3. Quadratic forms and self-adjoint operators [RS81, Vol. II, §X.2, Thm. X.17, and §X.3, Thm. X.23 and Example 1 on p. 178].
4. Spectrum of symmetric operators [RS81, Vol. II, Thm. X.1 and Corollaries at page 137].
5. Von Neumann theorem on self-adjoint extensions [RS81, Vol. II, Thm. X.2, Example 1 on p. 141, Thm. X.3].
6. Scattering theory: definition of scattering states and bound states, and their characterization [BEH08, pp. 491-496, §15.1 up to Prop. 15.1.4].
7. Scattering theory: definition of wave operators and their basic properties [BEH08, §15.2 up to Prop. 15.2.3].
8. Scattering theory: definition of wave operators and Cook's criterion for the existence of the wave operators [BEH08, Thm. 15.2.4, Thm. 15.2.5, Thm. 15.3.1].
9. Point interactions and approximation with regular potentials [BEH08, §14.6].
10. Birman–Schwinger principle, Birman–Schwinger bound, existence of bound states [BEH08, Lemma 14.3.3, Thm. 14.3.5, Thm. 14.3.6].
11. Riesz projection and discrete spectrum [RS81, Vol. IV, Theorem XII.5 and Theorem XII.6].
12. Point interaction in dimension $d = 1$: self-adjointness and scattering phenomena [AGHH88, Ch.1.3], [Te18, Ch.8].
13. Quantum waveguides [BEH08, Ch.16], [EK15, Ch.1].
14. Quantum graphs [BEH08, Ch.17], [EK15, Ch.8].
15. Algebraic formulation of Quantum Mechanics and the Gelfand–Naimark–Segal theorem [Mo17, Ch.14], [Mo19, Ch.8], [St08, Ch.2].

References

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[Te18] A. Teta, *A Mathematical Primer on Quantum Mechanics*, Springer (2018).